

Density-density propagator for one-dimensional interacting spinless fermions with non-linear dispersion and calculation of the Coulomb drag resistivity

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Using bosonization-fermionization transformation we map the Tomonaga-Luttinger model of spinless fermions with non-linear dispersion on the model of fermionic quasiparticles whose interaction is irrelevant in the renormalization group sense. Such mapping allows us to set up an expansion for the density-density propagator of the original Tomonaga-Luttinger Hamiltonian in orders of the (irrelevant) quasiparticle interaction. The lowest order term in such an expansion is proportional to the propagator for free fermions. The next term is also evaluated. The propagator found is used for calculation of the Coulomb drag resistivity r in a system of two capacitively coupled one-dimensional conductors. It is shown that r is proportional to T^2 for both free and interacting fermions. The marginal repulsive in-chain interaction acts to reduce r as compared to the non-interacting result. The correction to r due to the quasiparticle interaction is found as well. It scales as T^4 at low temperature.

I. INTRODUCTION

The bosonization has been an indispensable tool for one-dimensional (1D) interacting fermion studies. The advantage of the bosonization is that it allows to treat exactly the fermion interaction operator. This operator is marginal in the renormalization group (RG) sense, and therefore cannot be dealt with the help of perturbative approximations.

However the bosonization becomes inconvenient when one needs to go beyond the marginal operators. For example, when calculating the Coulomb drag resistivity in the system of two 1D wires [1, 2, 3, 4], it is necessary to account for the dispersion curvature v_F' of the fermions [5] which is an irrelevant operator. In the language of the bosonization such operator introduces interaction between the bosons with the coupling constant v_F' . This destroys the solubility of the bosonized Hamiltonian.

Moreover the Coulomb drag resistivity r is non-analytical in v_F' :

$$r \propto |v_F'|. \quad (1)$$

This indicates that simple perturbation theory in orders of v_F' is not applicable.

Different methods was used to address the issue of the non-linear dispersion [5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. Unfortunately, these papers either rely on numerical calculation or exact solubility or employ uncontrollable approximations or devise methods suitable for a particular task at hand. No universal approach emerged from those works.

A detour around the bosonization was proposed in [15, 16] where it was shown that a generic Tomonaga-Luttinger (TL) model of 1D interacting spinless fermions may be mapped on a system of free fermionic quasiparticles with weak irrelevant (in RG sense) interactions.

The latter approach was particularly convenient for evaluation of the density-density correlation function. It was demonstrated that this correlation function is proportional to the density-density correlation function of the free fermions plus small corrections due to the interactions between the quasiparticles. In Ref. [15, 16] the density-density retarded propagator $D_{k\omega}$ and the density spectral function $B_{k\omega} = -2\text{Im}D_{k\omega}$ were determined to zeroth order in the quasiparticle interaction.

Since the Coulomb drag resistivity is a functional of $B_{k\omega}$, it is natural to apply the method of [15, 16] to the problem of calculating the Coulomb drag resistivity. This is the purpose of this paper. More specifically, we show below that at small temperature $r \approx aT^2$, where the coefficient a is a decreasing function of the in-chain repulsion, and that correction due to the quasiparticle interactions δr vanishes quicker than T^2 : $\delta r \propto T^4$. These are two main results derived below.

This paper corrects [17] and the previous version of this preprint [18]. In these two publications the zero-temperature form of the spectral function is used for calculation of the finite-temperature drag. This mistake is not crucial for it does not alter neither T^2 behavior of r , nor T^4 behavior of δr . It does change the $O(1)$ numerical coefficient in front of formula for r , as well as details of r and δr derivations.

The presentation is structured as follows. First in Sect. II we establish the mapping of the TL model on the quasiparticle model. Next in Sect. III we obtain the density-density propagator and the Coulomb drag resistivity to zeroth and first order in the quasiparticle interaction. The conclusions of the paper are given in Sect. IV. Involved calculations are performed in Appendices.

II. THE MAPPING

In this paper we study the model of 1D fermions whose Hamiltonian is:

$$H = H_{\text{kin}} + H_{\text{nl}} + H_{\text{int}}, \quad (2)$$

$$H_{\text{kin}} = i v_F \int dx \left(: \psi_L^\dagger \nabla \psi_L : - : \psi_R^\dagger \nabla \psi_R : \right), \quad (3)$$

$$H_{\text{nl}} = v_F' \int dx \left(: (\nabla \psi_L^\dagger) (\nabla \psi_L) : + : (\nabla \psi_R^\dagger) (\nabla \psi_R) : \right), \quad (4)$$

$$H_{\text{int}} = g \int dx \rho_R \rho_L, \quad (5)$$

where $\psi_{R,L}$ are chiral fermionic fields corresponding to the right-moving (subscript 'R') and left-moving (subscript 'L') fermions, $\rho_{R,L} = : \psi_{R,L}^\dagger \psi_{R,L} :$ are chiral fermion densities, and the colons denote normal ordering. The cutoff Λ is assumed for this quantum field theory.

A non-perturbative approach to handle Hamiltonian H was proposed in Ref. [15]. There a unitary operator U was constructed which transforms H into the quasiparticle Hamiltonian:

$$H_{\text{qp}} = \tilde{H}_{\text{kin}} + \tilde{H}_{\text{nl}} + \tilde{H}_{\text{int}}' + \delta\mu(N_R + N_L), \quad (6)$$

where $N_{R,L}$ are the total number of right-moving (left-moving) fermions, \tilde{H}_{kin} and \tilde{H}_{nl} have the same form as H_{kin} and H_{nl} but with \tilde{v}_F and \tilde{v}_F' instead of v_F and v_F' :

$$\tilde{H}_{\text{kin}} = i \tilde{v}_F \int dx \left(: \tilde{\psi}_L^\dagger \nabla \tilde{\psi}_L : - : \tilde{\psi}_R^\dagger \nabla \tilde{\psi}_R : \right), \quad (7)$$

$$\tilde{H}_{\text{nl}} = \tilde{v}_F' \int dx \left(: (\nabla \tilde{\psi}_L^\dagger) (\nabla \tilde{\psi}_L) : + : (\nabla \tilde{\psi}_R^\dagger) (\nabla \tilde{\psi}_R) : \right), \quad (8)$$

$$\tilde{v}_F = v_F \sqrt{1 - \left(\frac{g}{2\pi v_F} \right)^2}, \quad (9)$$

$$\tilde{v}_F' = \frac{v_F'}{4} \left(\mathcal{K}^{3/2} + 3\mathcal{K}^{-1/2} \right). \quad (10)$$

The usual TL parameter \mathcal{K} is used in the last formula: $\mathcal{K} = \sqrt{(2\pi v_F - g)/(2\pi v_F + g)}$. Operators with tildes correspond to the quasiparticles: $\tilde{\psi}_p$ is the quasiparticle field, below we use $\tilde{\rho}_p$, which is the quasiparticle density.

The quasiparticle interaction \tilde{H}_{int}' in Eq.(6) is given by expression:

$$\tilde{H}_{\text{int}}' = - \sum_p i p \tilde{g}' \int dx \tilde{\rho}_{-p} \left[: \tilde{\psi}_p^\dagger (\nabla \tilde{\psi}_p) : - : (\nabla \tilde{\psi}_p^\dagger) \tilde{\psi}_p : \right] \quad (11)$$

$$\tilde{g}' = \frac{\pi v_F'}{2} \left(\mathcal{K}^{3/2} - \mathcal{K}^{-1/2} \right) \quad (12)$$

In Eq.(11) the summation runs over the chirality index $p = R, L$ whose numerical values are $p = +1$ for 'R' and $p = -1$ for 'L'. Observe that the operator \tilde{H}_{int}' is irrelevant: its scaling dimension is equal to 3 which is greater than 2 [19]. (To evaluate the scaling dimension one has to add together the scaling dimension of two

fermion operators, the gradient, and the electron density operator: $1/2 + 1/2 + 1 + 1 = 3$).

The chemical potential shift $\delta\mu$ in Eq.(6) is also induced by the transformation U . However since such shift causes nothing but additional renormalization of the quasiparticle dispersion parameters \tilde{v}_F and \tilde{v}_F' , we do not keep track of it below.

The readers who are interested to learn how operator U is constructed should consult Ref.[15, 16]. Here we use bosonization-fermionization sequence to establish the desired equivalence between H and H_{qp} . That way, we derive the result we need with no reference to the new technique unfamiliar to the majority of the researchers in the field.

The bosonization prescription [19, 20] expresses the 1D chiral fermion field as an exponential of the Bose field Φ and its conjugate Θ :

$$\psi_p^\dagger(x) = (2\pi a)^{-1/2} \eta_p e^{i\sqrt{\pi}[\Theta(x) + p\Phi(x)]}, \quad (13)$$

where $a \propto 1/\Lambda$ and η_p is the Klein factor. Consequences of this formula are:

$$\rho_p = \frac{1}{2\sqrt{\pi}} (\nabla\Phi + p\nabla\Theta), \quad (14)$$

$$-ip: \psi_p^\dagger (\nabla \psi_p) : + \text{H.c.} = \frac{1}{2} : (\nabla\Phi + p\nabla\Theta)^2 :, \quad (15)$$

$$: (\nabla \psi_p^\dagger) (\nabla \psi_p) : - \frac{1}{6} \nabla^2 \rho_p = \frac{\sqrt{\pi}}{6} : (\nabla\Phi + p\nabla\Theta)^3 :. \quad (16)$$

Using the above formulas we can write the bosonic form of H :

$$H[\Phi, \Theta] = H_{\text{kin}}[\Phi, \Theta] + H_{\text{int}}[\Phi, \Theta] + H_{\text{nl}}[\Phi, \Theta] \quad (17)$$

$$H_{\text{kin}} + H_{\text{int}} = \frac{\tilde{v}_F}{2} \int dx \left[\mathcal{K} : (\nabla\Theta)^2 : + \mathcal{K}^{-1} : (\nabla\Phi)^2 : \right] \quad (18)$$

$$H_{\text{nl}} = \frac{\sqrt{\pi}}{6} v_F' \int dx \sum_p : (\nabla\Phi + p\nabla\Theta)^3 : \quad (19)$$

Eq.(19) was derived in Ref. [21].

Once the bosonic form is explicitly written we are ready for the second step of the derivation – rescaling of the bosonic fields: $\tilde{\Phi} = \mathcal{K}^{-1/2}\Phi$, $\tilde{\Theta} = \mathcal{K}^{1/2}\Theta$. The different pieces of Eq. (17) can be expressed in terms of this new boson as such:

$$H_{\text{kin}} + H_{\text{int}} = \frac{\tilde{v}_F}{2} \int dx \left[: (\nabla\tilde{\Theta})^2 : + : (\nabla\tilde{\Phi})^2 : \right], \quad (20)$$

$$H_{\text{nl}} = \frac{\sqrt{\pi}}{6} v_F' \int dx \sum_p : \left(\mathcal{K}^{1/2} \nabla\tilde{\Phi} + p\mathcal{K}^{-1/2} \nabla\tilde{\Theta} \right)^3 :. \quad (21)$$

Introducing a new fermion (the quasiparticle) with the help of the formula:

$$\tilde{\psi}_p^\dagger(x) = (2\pi a)^{-1/2} \eta_p e^{i\sqrt{\pi}[\tilde{\Theta}(x) + p\tilde{\Phi}(x)]}, \quad (22)$$

one can refermionize H . Namely, inverting Eq.(15), we obtain for the sum $H_{\text{kin}} + H_{\text{int}}$:

$$H_{\text{kin}} + H_{\text{int}} = \tilde{H}_{\text{kin}}, \quad (23)$$

where \tilde{H}_{kin} is given by Eq.(7). On the right-hand side of this expression the interaction term $\int dx \tilde{\rho}_R \tilde{\rho}_L$ is absent. Thus, the marginal interaction, the most troublesome part of the Hamiltonian, is removed.

The price we have to pay for the absence of the marginal interaction is that H_{nl} expressed in $\tilde{\Phi}$, $\tilde{\Theta}$, Eq.(21), cannot be easily fermionized. It is convenient to rewrite the latter equation:

$$H_{\text{nl}} = \frac{\sqrt{\pi}}{6} \tilde{v}'_F \int dx \sum_p : (\nabla \tilde{\Phi} + p \nabla \tilde{\Theta})^3 : \quad (24)$$

$$+ \frac{\tilde{g}'}{4\sqrt{\pi}} \int dx \sum_p : (\nabla \tilde{\Phi} + p \nabla \tilde{\Theta})^2 (\nabla \tilde{\Phi} - p \nabla \tilde{\Theta}) :.$$

The advantage of this form is that it depends on $(\nabla \tilde{\Phi} \pm \nabla \tilde{\Theta})$ combinations only. Thus Eqs.(14), (15) and (16) may be immediately applied and H_{nl} may be fermionized:

$$H_{\text{nl}} = \tilde{H}_{\text{nl}} + \tilde{H}'_{\text{int}}, \quad (25)$$

where \tilde{H}_{nl} and \tilde{H}'_{int} are given by Eqs. (8) and (11).

This almost concludes the derivation of Eq.(6). What we lack is the $\delta\mu$ term of H_{qp} . To obtain this term we must (i) handle normal ordered expressions more accurately and (ii) take special care about the zero modes $N_{\text{R,L}}$. Technically this is similar to the treatment of Ref. [15, 16]. Since we are not interested in the chemical potential shift, we do not address this issue here.

III. DENSITY-DENSITY PROPAGATOR AND THE COULOMB DRAG RESISTIVITY

Once the mapping of H on H_{qp} is established, we can use it to calculate the density-density propagator $D_{k\omega}$. First let us find the following density-density correlation function:

$$\mathcal{R} = \langle [\rho_{\text{R}}(x, \tau) + \rho_{\text{L}}(x, \tau)] [\rho_{\text{R}}(0, 0) + \rho_{\text{L}}(0, 0)] \rangle. \quad (26)$$

In the bosonic form it equals to:

$$\mathcal{R} = \frac{1}{\pi} \langle \nabla \tilde{\Phi}(x) e^{-\tau H} \nabla \tilde{\Phi}(0) \rangle. \quad (27)$$

After the field rescaling it becomes:

$$\mathcal{R} = \frac{\mathcal{K}}{\pi} \langle \nabla \tilde{\Phi}(x) e^{-\tau H_{\text{qp}}} \nabla \tilde{\Phi}(0) \rangle. \quad (28)$$

Under fermionization \mathcal{R} transforms into:

$$\mathcal{R} = \mathcal{K} \langle [\tilde{\rho}_{\text{R}}(x, \tau) + \tilde{\rho}_{\text{L}}(x, \tau)] [\tilde{\rho}_{\text{R}}(0, 0) + \tilde{\rho}_{\text{L}}(0, 0)] \rangle_{\text{qp}}, \quad (29)$$

where the subscript ‘qp’ reminds that the averaging is to be performed with respect to the quasiparticle Hamiltonian H_{qp} .

One can prove through the same line of reasoning that

$$D_{k\omega} = \mathcal{K} \tilde{D}_{k\omega}, \quad (30)$$

where $\tilde{D}_{k\omega}$ is the retarded density-density propagator for the quasiparticle Hamiltonian H_{qp} . Consequently, the task of finding the density-density propagator for the physical fermions is reduced to the task of finding the quasiparticle density-density propagator. The latter is much easier, for the quasiparticle interaction \tilde{H}'_{int} is irrelevant in the RG sense.

As a starting point we calculate $D_{k\omega}$ to zeroth order in \tilde{H}'_{int} :

$$D_{k\omega} = \mathcal{K} (\tilde{D}_{\text{R}k\omega}^0 + \tilde{D}_{\text{L}k\omega}^0) + O(\tilde{H}'_{\text{int}}), \quad (31)$$

$$B_{k\omega} = \mathcal{K} (\tilde{B}_{\text{R}k\omega}^0 + \tilde{B}_{\text{L}k\omega}^0) + O(\tilde{H}'_{\text{int}}), \quad (32)$$

where the chiral quasiparticle density-density propagators $\tilde{D}_{\text{R,L}}^0$ and corresponding spectral densities $\tilde{B}_{\text{R,L}}^0$ are:

$$\tilde{D}_{pk\omega}^0 = \int_q \frac{n_{pq-k/2} - n_{pq+k/2}}{\omega + \tilde{\varepsilon}_{pq-k/2} - \tilde{\varepsilon}_{pq+k/2} + i0}, \quad (33)$$

$$\tilde{B}_{pk\omega}^0 = \int_{-\Lambda}^{\Lambda} dq (n_{pq-k/2} - n_{pq+k/2}) \times \delta(\omega + \tilde{\varepsilon}_{pq-k/2} - \tilde{\varepsilon}_{pq+k/2}). \quad (34)$$

Symbol $\int_q \dots$ stands for $(2\pi)^{-1} \int_{-\Lambda}^{\Lambda} dq \dots$, quantity $\tilde{\varepsilon}_{pq}$ is the quasiparticle dispersion, and n_{pq} is the Fermi occupation number:

$$\tilde{\varepsilon}_{pq} = p \tilde{v}_{\text{F}} q + \tilde{v}'_{\text{F}} q^2, \quad (35)$$

$$n_{pq} = [1 + \exp(\tilde{\varepsilon}_{pq}/T)]^{-1}. \quad (36)$$

At $T = 0$ the integrals in Eq. (33) and Eq. (34) can be easily evaluated. One finds the lowest order expression for D and B [15, 16]:

$$D_{k\omega} = \frac{\mathcal{K}}{4\pi \tilde{v}'_{\text{F}} k} \ln \left[\frac{(\tilde{v}_{\text{F}} k - \tilde{v}'_{\text{F}} k^2)^2 - (\omega + i0)^2}{(\tilde{v}_{\text{F}} k + \tilde{v}'_{\text{F}} k^2)^2 - (\omega + i0)^2} \right] + O(\tilde{H}'_{\text{int}}), \quad (37)$$

$$B_{k\omega} = \frac{\mathcal{K}}{2\tilde{v}'_{\text{F}} k} [\vartheta(\omega^2 - (\tilde{v}_{\text{F}} k - \tilde{v}'_{\text{F}} k^2)^2) - \vartheta(\omega^2 - (\tilde{v}_{\text{F}} k + \tilde{v}'_{\text{F}} k^2)^2)] \text{sgn } \omega + O(\tilde{H}'_{\text{int}}). \quad (38)$$

For non-zero temperature Aristov shows [13] that Eq. (33) reduces to an expression with a special function. Eq. (34) may be calculated even for non-zero temperature in terms of elementary functions:

$$\tilde{B}_{pk\omega}^0 = \frac{\sinh(\frac{\omega}{2T})}{4|\tilde{v}'_{\text{F}} k| \cosh(\tilde{v}_{\text{F}} \frac{\delta\omega_p - \tilde{v}'_{\text{F}} k^2}{4\tilde{v}'_{\text{F}} k T}) \cosh(\tilde{v}_{\text{F}} \frac{\delta\omega_p + \tilde{v}'_{\text{F}} k^2}{4\tilde{v}'_{\text{F}} k T})}, \quad (39)$$

$$\delta\omega_p = \omega - p \tilde{v}_{\text{F}} k. \quad (40)$$

Appendix A provides the details of the calculation. Eq. (39) can be used to find $B_{k\omega}$, Eq. (32).

Once the spectral function is found the Coulomb drag resistivity may be evaluated with its help. Before proceeding with such calculations, let us briefly explain what Coulomb drag is.

In the Coulomb drag experiment two parallel 1D wires (subscript $i = 1, 2$) of length L with Hamiltonians H , Eq.(2), are coupled capacitively with the Hamiltonian $H_C = g_C \int dx \rho_1 \rho_2$. Because of this coupling, electrical current I in one of the wires induces potential drop V across the other wire. The proportionality coefficient between V and I is called the Coulomb drag resistivity $r = V/IL$. It characterizes the pulling force, which the fermions in the current-carrying wire exert on the fermions in the other wire.

The experimental physics of the Coulomb drag is quite rich: the observed values of r could be either positive or negative, and show dependence on temperature, spacial inhomogeneity, and applied magnetic field.

In general, two mechanisms are discussed in the theoretical literature [3]. According to one mechanism, the drag occurs because Wigner crystal-like correlations in both wires lock against each other. In the RG language this corresponds to the relevance of the inter-wire backscattering interaction. Such mechanism works best at zero temperature but quickly deteriorates at $T > 0$. We do not study this mechanism in our paper.

The second mechanism is insensitive to the inter-wire backscattering. Instead it relies upon interaction of the smooth components of the electron densities. It is more resilient towards temperature but the resultant value of r is proportional to $|v'_F|$. Because of this, it is impossible to study the second mechanism within the well-established framework of the one-dimensional bosonization. The purpose of this paper is to provide a reliable approach overcoming this difficulty.

We start with the following formula for r (Eq.(7) of Ref. [5]) which is valid when the inter-wire backscattering can be neglected:

$$r = \frac{g_C^2}{16\pi^3 n^2 T} \int_0^{+\infty} dk \int_0^{+\infty} d\omega \frac{k^2 B_{1,k\omega} B_{2,k\omega}}{\text{sh}^2(\omega/2T)}. \quad (41)$$

Here n is the electron density. The spectral density $A(k, \omega)$ of Ref. [5] relates to our spectral density $B_{k\omega}$ as: $B_{k\omega} = -2A(k, \omega)$. In that reference the notation U_{12} is used for the inter-wire coupling constant g_C . Below we study the case of identical wires: $B_1 = B_2 = B$. We assume that $|\tilde{v}'_F|$ is small:

$$|\tilde{v}'_F| \Lambda < \tilde{v}_F. \quad (42)$$

Since we know $B_{k\omega}$ to zeroth order in \tilde{g}' , it is straightforward to find r with the same accuracy. Let us begin our calculation with the following observation. The function:

$$\begin{aligned} \tilde{b}_{pk\omega} &= \frac{\tilde{B}_{pk\omega}^0}{\sinh(\frac{\omega}{2T})} \\ &= \frac{1}{2|\tilde{v}'_F|k \left[\cosh(\frac{\tilde{v}_F \delta\omega_p}{2\tilde{v}'_F k T}) + \cosh(\frac{\tilde{v}_F k}{2T}) \right]} \end{aligned} \quad (43)$$

localized mostly at small momenta and near the 'light cone':

$$|k| < T/\tilde{v}_F, \quad (44)$$

$$|\omega - p\tilde{v}_F k| < \tilde{v}'_F k T / \tilde{v}_F < \tilde{v}'_F T^2 / \tilde{v}_F^2. \quad (45)$$

Outside of this region $\tilde{b}_{pk\omega}$ is exponentially small.

Since $\tilde{b}_{Lk\omega}$ is exponentially small when k and ω are positive, for the purpose of calculating the integral Eq. (41) we may further approximate: $B \approx \mathcal{K} \tilde{B}_R^0$. Substituting this in Eq. (41) we obtain:

$$r = \frac{g_C^2 \mathcal{K}^2}{256\pi^3 (\tilde{v}'_F)^2 n^2 T} I(T), \quad (46)$$

$$\begin{aligned} I(T) &= 16(\tilde{v}'_F)^2 \int_0^{+\infty} dk \int_0^{+\infty} d\omega k^2 \tilde{b}_{Rk\omega}^2 \\ &= \int \frac{dk d\omega}{\cosh^2(\tilde{v}_F \frac{\omega - \tilde{v}_F k - \tilde{v}'_F k^2}{4\tilde{v}'_F k T}) \cosh^2(\tilde{v}_F \frac{\omega - \tilde{v}_F k + \tilde{v}'_F k^2}{4\tilde{v}'_F k T})}. \end{aligned} \quad (47)$$

The integral $I(T)$ is evaluated in Appendix B. It is shown that, if temperature is low: $T \ll \tilde{\epsilon}_F$, where renormalized Fermi energy is:

$$\tilde{\epsilon}_F = \frac{\tilde{v}_F^2}{4\tilde{v}'_F}, \quad (48)$$

then

$$I = \frac{32|\tilde{v}'_F|T^3}{\tilde{v}_F^3} + o(T^3). \quad (49)$$

This gives us:

$$r = \frac{g_C^2 \mathcal{K}^2}{8\pi^3 \tilde{v}_F^3 |\tilde{v}'_F| n^2} T^2 + o(T^2). \quad (50)$$

The electron zero-temperature density, which enters Eq. (50), is not an independent quantity. It can be expressed in terms of the dispersion parameters: $n = v_F/(2\pi|v'_F|)$.

Furthermore, it is possible to show with the help of Eqs. (9) and (10) that $v_F = \tilde{v}_F + O(g^2)$ and $v'_F = \tilde{v}'_F + O(g^2)$. Since our accuracy does not allow us to keep $O(g^2)$ terms, we can replace \tilde{v}_F by v_F and \tilde{v}'_F by v'_F in Eq.(50). Thus, it is true:

$$r \approx a T^2, \quad (51)$$

$$a = \frac{g_C^2 \mathcal{K}^2 |v'_F|}{2\pi v_F^5} + O(g^2). \quad (52)$$

The interaction enters the expression for a through $\mathcal{K}^2 = 1 - g/(\pi v_F) + O(g^2)$. We see that the repulsive in-chain interaction ($g > 0$) acts to reduce r .

Eq.(51) may be cast in the following form:

$$r \approx \frac{c_1 \mathcal{K}^2}{l_0} \left(\frac{T}{\epsilon_F} \right)^2, \quad (53)$$

where l_0^{-1} and ϵ_F are defined in Ref. [5]. They are:

$$\epsilon_F = \tilde{\epsilon}_F + O(g^2), \quad (54)$$

$$\frac{1}{l_0} = \left(\frac{g_C}{2\pi v_F} \right)^2 n = \frac{g_C^2}{8\pi^3 |v'_F| v_F}. \quad (55)$$

The numerical coefficient c_1 is equal to $\pi^2/4$. For free fermions ($\mathcal{K} = 1$) the formula identical to Eq. (53), with the same value for c_1 is derived in [5].

Furthermore, that paper establishes that $r \propto T^2$ for both non-interacting fermions and for exactly soluble Calogero-Sutherland model. Our Eq.(50) proves that T^2 dependence holds for a generic interacting model as well.

In Refs. [17, 18], where, mistakenly, the zero-temperature form of B is used in the integral for r , different value of the coefficient c_1 is found. Present derivation corrects that error.

Eq.(51) and Eq. (52) account for effects of marginal interaction g . In addition to that, our method allows us to evaluate the correction δr to r due to the quasiparticle interaction \tilde{g}' .

To find δr we calculate the lowest order correction to $D_{k\omega}^0$. Such correction appears in the first order in \tilde{g}' : since \tilde{H}'_{int} couples the right-moving and the left-moving quasiparticles, the expectation value $\langle \tilde{\rho}_R(x, \tau) \tilde{\rho}_L(0, 0) \rangle$ taken with respect to H_{qp} , Eq.(6), is no longer zero but instead $O(\tilde{g}')$. Thus, full Matsubara propagator $\mathcal{D} = \mathcal{D}^0 + \delta\mathcal{D}$, where:

$$\begin{aligned} \delta\mathcal{D}(x, \tau) &= \mathcal{K} \delta\tilde{\mathcal{D}}(x, \tau) \\ &\approx \mathcal{K} \int d\tau' \left[\langle \tilde{\rho}_R(x, \tau) \tilde{H}'_{\text{int}}(\tau') \tilde{\rho}_L(0, 0) \rangle_0 \right. \\ &\quad \left. + \langle \tilde{\rho}_L(x, \tau) \tilde{H}'_{\text{int}}(\tau') \tilde{\rho}_R(0, 0) \rangle_0 \right]. \end{aligned} \quad (56)$$

Here the symbol $\langle \langle \dots \rangle \rangle_0$ stands for time-ordered averaging with respect to the non-interacting ($\tilde{g}' = 0$) quasiparticle Hamiltonian. Matsubara propagators are denoted by calligraphic letters (e.g., \mathcal{D} , \mathcal{P}), retarded propagators are denoted by italic letters (e.g., D , P).

Feynman diagram, which describes $\delta\tilde{\mathcal{D}}$, is shown on Fig.1. The wavy interaction line is to be identified with $\tilde{g}'(k_1 + k_2 - p_1 - p_2)$, k_i, p_i are the fermion momenta.

When evaluating $\delta\mathcal{D}$ we note that in the non-interacting quasiparticle Hamiltonian the left-moving and the right-moving quasiparticles are decoupled from each other. Consequently, the object $\langle \langle \rho_{L,R} H'_{\text{int}} \rho_{L,R} \rangle \rangle_0$ is split into products of left-only and right-only expectation values.

As a result of this derivation one can show that the lowest order correction to the Matsubara propagator due to the quasiparticle interaction is:

$$\delta\mathcal{D}_{k\omega} = -2\tilde{g}'\mathcal{K} \sum_p \tilde{\mathcal{D}}_{pk\omega}^0 \tilde{\mathcal{P}}_{-pk\omega}, \quad (57)$$

where $\tilde{\mathcal{D}}_{pk\omega}^0$ is the chiral non-interacting Matsubara propagator, the propagator $\tilde{\mathcal{P}}_p$ is defined as:

$$\begin{aligned} \tilde{\mathcal{P}}_p(x, \tau) &= -ip \langle \langle \tilde{\rho}_p(x, \tau) \\ &\quad \times \left\{ \tilde{\psi}_p^\dagger(0, 0) [\nabla \tilde{\psi}_p(0, 0)] : - : [\nabla \tilde{\psi}_p^\dagger(0, 0)] \tilde{\psi}_p(0, 0) : \right\} \rangle \rangle_0. \end{aligned} \quad (58)$$

It equals to:

$$\tilde{\mathcal{P}}_{pk\omega} = -\frac{1}{2\pi\tilde{v}'_F} + \frac{(i\omega - p\tilde{v}_F k)}{p\tilde{v}'_F k} \tilde{\mathcal{D}}_{pk\omega}^0. \quad (59)$$

To find the correction to the spectral function δB it is necessary to perform the analytic continuation in Eq.(57): $\delta D_{k\omega} = -2\tilde{g}'\mathcal{K} \sum_p \tilde{D}_{-pk\omega}^0 \tilde{P}_{pk\omega}$, where the retarded quantity $\tilde{P}_{pk\omega} = \tilde{\mathcal{P}}_{pk\omega}|_{i\omega \rightarrow \omega + i0}$ is equal to:

$$\tilde{P}_{pk\omega} = -\frac{1}{2\pi\tilde{v}'_F} + \frac{(\omega - p\tilde{v}_F k + i0)}{p\tilde{v}'_F k} \tilde{D}_{pk\omega}^0. \quad (60)$$

We obtain for δB :

$$\delta B_{k\omega} = \frac{\tilde{g}'\mathcal{K}}{\pi\tilde{v}'_F} \sum_p \tilde{B}_{-pk\omega}^0 \left(4\pi\tilde{v}_F \text{Re} \tilde{D}_{pk\omega}^0 + 1 \right). \quad (61)$$

This is the lowest order correction to the spectral density due to the quasiparticle interactions.

The square of the spectral density, which enters the equation for the Coulomb drag, is:

$$\begin{aligned} B_{k\omega}^2 &\approx (\mathcal{K} \tilde{B}_{k\omega}^0)^2 \\ &+ \frac{2\tilde{g}'\mathcal{K}^2}{\pi\tilde{v}'_F} \sum_{p,p'} \tilde{B}_{p'k\omega}^0 \tilde{B}_{-pk\omega}^0 \left(4\pi\tilde{v}_F \text{Re} \tilde{D}_{pk\omega}^0 + 1 \right). \end{aligned} \quad (62)$$

We already pointed out the fact that $\tilde{b}_{Lk\omega}$ is exponentially small for positive k and ω . Therefore, in the region of the integration we may neglect the terms proportional to $\tilde{B}_L^0 \tilde{B}_R^0$ and $(\tilde{B}_L^0)^2$:

$$B_{k\omega}^2 \approx (\mathcal{K} \tilde{B}_{Rk\omega}^0)^2 + \frac{2\tilde{g}'\mathcal{K}^2}{\pi\tilde{v}'_F} (\tilde{B}_{Rk\omega}^0)^2 \left(4\pi\tilde{v}_F \text{Re} \tilde{D}_{Lk\omega}^0 + 1 \right) \quad (63)$$

The correction to the drag is:

$$\delta r = \frac{g_C^2 \tilde{g}'\mathcal{K}^2}{8\pi^2 \tilde{v}'_F \tilde{v}_F T} \int \frac{dk d\omega \left(4\pi\tilde{v}_F \text{Re} \tilde{D}_{Lk\omega}^0 + 1 \right)}{\left[\cosh\left(\frac{\tilde{v}_F \delta\omega_R}{2\tilde{v}'_F k T}\right) + \cosh\left(\frac{\tilde{v}_F k}{2T}\right) \right]^2}. \quad (64)$$

Since the denominator in this integral is very small everywhere except near $\omega = \tilde{v}_F k$, we need to know the behavior of $\text{Re} \tilde{D}_L^0$ in this region. It is proven in Appendix C that:

$$\begin{aligned} 4\pi\tilde{v}_F \text{Re} \tilde{D}_{Lk\omega}^0 + 1 &= \frac{\delta\omega_R}{2\tilde{v}_F k} - \frac{1}{12} \left(\frac{\tilde{v}'_F k}{\tilde{v}_F} \right)^2 \\ &- \frac{1}{4} \left(\frac{\delta\omega_R}{\tilde{v}_F k} \right)^2 - \frac{\pi^2 T^2}{24 \tilde{v}_F^2} + O(T^3), \end{aligned} \quad (65)$$

where it is assumed that $\tilde{v}_F k = O(T)$, and $\tilde{v}_F \delta\omega_R / \tilde{v}'_F k = O(T)$.

At $T = 0$ the above expansion becomes:

$$4\pi\tilde{v}_F \text{Re} \tilde{D}_{pk\omega}^0 + 1 = -\frac{p\delta\omega_p}{2\tilde{v}_F k} - \frac{(\delta\omega_p)^2}{4\tilde{v}_F^2 k^2} - \frac{(\tilde{v}'_F k)^2}{12\tilde{v}_F^2}. \quad (66)$$

This expansion is derived in [17, 18], see Eq. (51) of these references.

Finally, placing Eq. (65) into Eq. (64) one finds

$$\delta r \propto T^4. \quad (67)$$

This result may be obtained without performing actual integration. It is enough to use the dimensional analysis:

$$4\pi\tilde{v}_F\text{Re}\tilde{D}_{pk\omega}^0 + 1 = O(T^2), \quad (68)$$

$$dk = O(T), \quad (69)$$

$$d\omega = O(\delta\omega_R) = O(T^2). \quad (70)$$

From these Eq. (67) follows.

Consequently, at low temperature the interaction correction δr to the Coulomb drag resistivity r vanishes quicker than T^2 . Thus, the non-interacting quasiparticle result, Eq.(50), suffices to capture the leading behavior of r at $T \rightarrow 0$.

IV. CONCLUSIONS

Using bosonization-fermionization trick we mapped the Tomonaga-Luttinger Hamiltonian with non-linear dispersion on the Hamiltonian of the quasiparticles with irrelevant interaction. This mapping allows us to evaluate the density-density propagator of the Tomonaga-Luttinger model with non-linear dispersion. The propagator itself was used to calculate the temperature dependence of the Coulomb drag resistivity r . It was established that $r \propto T^2$ at low T for both interacting and free fermions. The irrelevant quasiparticle interaction introduces additional correction which vanishes as T^4 .

V. ACKNOWLEDGEMENTS

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APPENDIX A: SOME DETAILS OF $\tilde{B}_{pk\omega}^0$ CALCULATION

Here we fill the gap between Eq. (34) and Eq. (39). Observe that:

$$n_{pq-k/2} - n_{pq+k/2} = \frac{\sinh\left(\frac{\tilde{\varepsilon}_{pq+k/2} - \tilde{\varepsilon}_{pq-k/2}}{2T}\right)}{2 \cosh\left(\frac{\tilde{\varepsilon}_{pq-k/2}}{2T}\right) \cosh\left(\frac{\tilde{\varepsilon}_{pq+k/2}}{2T}\right)} \quad (A1)$$

The delta-function in Eq. (34) enforces $\tilde{\varepsilon}_{pq+k/2} - \tilde{\varepsilon}_{pq-k/2} = \omega$. Therefore:

$$\begin{aligned} \tilde{B}_{pk\omega}^0 &= \int dq \frac{\sinh\left(\frac{\omega}{2T}\right) \delta(\omega - p\tilde{v}_F k - 2\tilde{v}'_F k q)}{2 \cosh\left(\frac{\tilde{\varepsilon}_{pq-k/2}}{2T}\right) \cosh\left(\frac{\tilde{\varepsilon}_{pq+k/2}}{2T}\right)} \quad (A2) \\ &= \frac{\sinh\left(\frac{\omega}{2T}\right)}{2|\tilde{v}'_F k|} \int dq \frac{\delta\left(\frac{\omega - p\tilde{v}_F k}{2\tilde{v}'_F k} - q\right)}{2 \cosh\left(\frac{\tilde{\varepsilon}_{pq-k/2}}{2T}\right) \cosh\left(\frac{\tilde{\varepsilon}_{pq+k/2}}{2T}\right)}. \end{aligned}$$

Substituting $q = (\omega - p\tilde{v}_F k)/2\tilde{v}'_F k$, as required by the delta-function, into expressions for $\tilde{\varepsilon}_{pq+k/2}$ we obtain:

$$q + k/2 = \frac{\delta\omega_p + \tilde{v}'_F k^2}{2\tilde{v}'_F k}, \quad (A3)$$

$$\varepsilon_{pq+k/2} = \frac{(\delta\omega_p + \tilde{v}'_F k^2)(\delta\omega_p + 2p\tilde{v}_F k + \tilde{v}'_F k^2)}{4\tilde{v}'_F k^2}, \quad (A4)$$

$$\delta\omega_p = \omega - p\tilde{v}_F k. \quad (A5)$$

One can check that the right-hand side of Eq. (A4) vanishes both at $\delta\omega_p = -\tilde{v}'_F k^2$ and at $\delta\omega_p = 2\tilde{v}_F k - \tilde{v}'_F k^2$. Of these two roots only the first is physical, the second is spurious. The spurious root appears due to unjustified extrapolation of Eq. (35) to large momenta $\sim \tilde{v}_F/\tilde{v}'_F$. Indeed, using Eq. (A3), we find that the momentum $q+k/2$ is close to zero for the physical root and close to $-p\tilde{v}_F/\tilde{v}'_F$ for spurious root. Since our treatment is valid near the Fermi points only we replace:

$$\delta\omega_p + 2p\tilde{v}_F k + \tilde{v}'_F k^2 \approx 2p\tilde{v}_F k. \quad (A6)$$

In the above equation we neglected the terms, which are quadratic in T , see Eq. (45). Consequently:

$$\tilde{\varepsilon}_{pq+k/2} \approx \frac{\tilde{v}_F}{2p\tilde{v}'_F k}(\delta\omega_p + \tilde{v}'_F k^2). \quad (A7)$$

With the help of the same arguments one derives:

$$\tilde{\varepsilon}_{pq-k/2} \approx \frac{\tilde{v}_F}{2p\tilde{v}'_F k}(\delta\omega_p - \tilde{v}'_F k^2). \quad (A8)$$

The expressions Eq. (A7) and Eq. (A8) allow us to perform integration over q in Eq. (A2):

$$\tilde{B}_{pk\omega}^0 = \frac{\sinh\left(\frac{\omega}{2T}\right)}{4|\tilde{v}'_F k| \cosh\left(\tilde{v}_F \frac{\delta\omega_p - \tilde{v}'_F k^2}{4\tilde{v}'_F k T}\right) \cosh\left(\tilde{v}_F \frac{\delta\omega_p + \tilde{v}'_F k^2}{4\tilde{v}'_F k T}\right)} \quad (A9)$$

This is the quasiparticle spectral function for $T > 0$.

APPENDIX B: CALCULATION OF $I(T)$

The integral $I(T)$ is defined as:

$$I(T) = \int_0^{+\infty} dk \int_0^{+\infty} d\omega \times \frac{1}{\cosh^2\left(\tilde{v}_F \frac{\omega - \tilde{v}_F k - \tilde{v}'_F k^2}{4\tilde{v}'_F k T}\right) \cosh^2\left(\tilde{v}_F \frac{\omega - \tilde{v}_F k + \tilde{v}'_F k^2}{4\tilde{v}'_F k T}\right)}. \quad (B1)$$

In the above integral we introduce the new variable:

$$\Omega = \frac{\tilde{v}_F}{4|\tilde{v}'_F|kT}(\omega - \tilde{v}_F k - |\tilde{v}'_F|k^2), \quad (B2)$$

$$I = \frac{4|\tilde{v}'_F|T}{\tilde{v}_F} \int_0^{+\infty} dk \int_{-\Omega_0}^{+\infty} \frac{k d\Omega}{\cosh^2 \Omega \cosh^2\left(\Omega + \frac{\tilde{v}_F k}{2T}\right)}, \quad (B3)$$

$$\Omega_0 = \frac{\tilde{v}_F^2}{4|\tilde{v}'_F|T} + \frac{\tilde{v}_F k}{4T}. \quad (B4)$$

If temperature is low: $T \ll \tilde{\epsilon}_F$, where

$$\tilde{\epsilon}_F = \frac{\tilde{v}_F^2}{4\tilde{v}_F'}, \quad (\text{B5})$$

then $\Omega_0 \gg 1$. In such a situation the integrand is exponentially small for $\Omega < -\Omega_0$:

$$\cosh^{-2} \Omega \cosh^{-2}(\Omega + \tilde{v}_F k/2T) \leq \cosh^{-2} \Omega < 4 \exp(-2\Omega_0) \ll 1. \quad (\text{B6})$$

Therefore, we may extend the integration interval:

$$\int_{-\Omega_0}^{+\infty} d\Omega \dots \approx \int_{-\infty}^{+\infty} d\Omega \dots \quad (\text{B7})$$

The integral I can be expressed as:

$$I = \frac{4|\tilde{v}_F'|T}{\tilde{v}_F} \int_0^{+\infty} k dk \int_{-\infty}^{+\infty} \frac{d\Omega}{\cosh^2 \Omega \cosh^2(\Omega + \frac{\tilde{v}_F k}{2T})} \quad (\text{B8})$$

$$= \frac{16|\tilde{v}_F'|T^3}{\tilde{v}_F^3} \int_{-\infty}^{+\infty} \frac{d\Omega}{\cosh^2 \Omega} \int_0^{+\infty} \frac{Q dQ}{\cosh^2(\Omega + Q)}.$$

Integrating over Q by parts we derive:

$$I(T) = \frac{16|\tilde{v}_F'|T^3}{\tilde{v}_F^3} \int_{-\infty}^{+\infty} \frac{d\Omega}{\cosh^2 \Omega} f(\Omega), \quad (\text{B9})$$

$$f(\Omega) = \int_0^{+\infty} [1 - \tanh(\Omega + Q)] dQ. \quad (\text{B10})$$

Since $f(\Omega) \rightarrow 0$ when $\Omega \rightarrow +\infty$, we can integrate by parts over Ω as follows:

$$I(T) = -\frac{16|\tilde{v}_F'|T^3}{\tilde{v}_F^3} \int_{-\infty}^{+\infty} (\tanh \Omega + 1) f'(\Omega) d\Omega, \quad (\text{B11})$$

$$f'(\Omega) = -\int_0^{+\infty} \frac{d\Omega}{\cosh^2(\Omega + Q)} = \tanh \Omega - 1. \quad (\text{B12})$$

Therefore:

$$I(T) = \frac{16|\tilde{v}_F'|T^3}{\tilde{v}_F^3} \int_{-\infty}^{+\infty} (1 - \tanh^2 \Omega) d\Omega. \quad (\text{B13})$$

The last integral can be calculated easily, and $I(T)$ is:

$$I(T) = \frac{32|\tilde{v}_F'|T^3}{\tilde{v}_F^3}. \quad (\text{B14})$$

This concludes our derivation of $I(T)$.

APPENDIX C: EXPANSION OF $\text{Re} \tilde{D}_L$

In this Appendix we evaluate $\text{Re} \tilde{D}_{Lk\omega}^0$ when $\omega \approx \tilde{v}_F k$. We start with the following expression:

$$\text{Re} \tilde{D}_{Lk\omega}^0 = \text{Re} \int_q \frac{n_{Lq-k/2} - n_{Lq+k/2}}{\omega + \tilde{\epsilon}_{Lq-k/2} - \tilde{\epsilon}_{Lq+k/2} + i0}. \quad (\text{C1})$$

From which one gets:

$$\text{Re} \tilde{D}_{Lk\omega}^0 = -\frac{1}{2\tilde{v}_F' k} \text{Re} \int_q \frac{n_{Lq-k/2} - n_{Lq+k/2}}{q - \delta\omega_L/2\tilde{v}_F' k + i0}. \quad (\text{C2})$$

In the last expression we integrate by parts to obtain:

$$\text{Re} \tilde{D}_{Lk\omega}^0 = \frac{1}{2\tilde{v}_F' k} \int_q \ln \left| \frac{q + \frac{k}{2} - \frac{\delta\omega_L}{2\tilde{v}_F' k}}{q - \frac{k}{2} - \frac{\delta\omega_L}{2\tilde{v}_F' k}} \right| \frac{dn_{Lq}}{dq}. \quad (\text{C3})$$

Since we want to derive $\text{Re} \tilde{D}_{Lk\omega}^0$ near the point $\delta\omega_R \approx 0$, it is convenient to substitute $\delta\omega_L$ with the equal quantity $\delta\omega_R + 2\tilde{v}_F k$. After such substitution and some transformations the expression for $\text{Re} \tilde{D}_{Lk\omega}^0$ becomes:

$$\text{Re} \tilde{D}_{Lk\omega}^0 = \frac{1}{2\tilde{v}_F' k} \int_q \ln \left| \frac{1 - \frac{\tilde{v}_F' q}{\tilde{v}_F} - \frac{\tilde{v}_F' k}{2\tilde{v}_F} + \frac{\delta\omega_R}{2\tilde{v}_F' k}}{1 - \frac{\tilde{v}_F' q}{\tilde{v}_F} + \frac{\tilde{v}_F' k}{2\tilde{v}_F} + \frac{\delta\omega_R}{2\tilde{v}_F' k}} \right| \frac{dn_{Lq}}{dq} \quad (\text{C4})$$

Logarithm in this formula may be expanded in orders of small parameter $T/\tilde{\epsilon}_F$. Indeed, it is true that $\tilde{v}_F |k| = O(T)$ and $\tilde{v}_F |\delta\omega_R/\tilde{v}_F' k| = O(T)$. Observe also:

$$\int q dq \frac{dn_{Lq}}{dq} = \int d\epsilon \left(\frac{\epsilon}{\tilde{v}_F} - \frac{\tilde{v}_F' \epsilon^2}{\tilde{v}_F^3} \right) \frac{d}{d\epsilon} \frac{1}{1 + \exp(\epsilon/T)} \quad (\text{C5})$$

$$= \frac{\tilde{v}_F'}{4\tilde{v}_F^3 T} \int d\epsilon \frac{\epsilon^2}{\cosh^2(\epsilon/2T)} = \frac{\pi^2 \tilde{v}_F' T^2}{3\tilde{v}_F^3},$$

where we used $q \approx -\tilde{\epsilon}_{Lq}/\tilde{v}_F + \tilde{v}_F' \tilde{\epsilon}_{Lq}/\tilde{v}_F^3$ and the identity:

$$\int_{-\infty}^{+\infty} \frac{x^2 dx}{\cosh^2 x} = \frac{\pi^2}{6}. \quad (\text{C6})$$

Using similar procedure one proves:

$$\int q^2 dq \frac{dn_{Lq}}{dq} = \frac{\pi^2 T^2}{3\tilde{v}_F^2}. \quad (\text{C7})$$

Therefore, both q and q^2 , upon integration, contribute terms of order T^2 . Thus:

$$\text{Re} \tilde{D}_{Lk\omega}^0 = -\frac{1}{2\tilde{v}_F} \int_q \frac{dn_{Lq}}{dq} \left[1 + \frac{\tilde{v}_F' q}{\tilde{v}_F} - \frac{\delta\omega_R}{2\tilde{v}_F' k} \right. \quad (\text{C8})$$

$$\left. + \frac{1}{12} \left(\frac{\tilde{v}_F' q}{\tilde{v}_F} \right)^2 + \left(\frac{\tilde{v}_F' k}{\tilde{v}_F} \right)^2 + \frac{1}{4} \left(\frac{\delta\omega_R}{\tilde{v}_F' k} \right)^2 \right] + O(T^3).$$

Since $\int_q dn_{Lq}/dq = 1/(2\pi)$, Eq. (C8) may be written as:

$$4\pi \tilde{v}_F \text{Re} \tilde{D}_{Lk\omega}^0 + 1 = \frac{\delta\omega_R}{2\tilde{v}_F k} - \frac{1}{12} \left(\frac{\tilde{v}_F' k}{\tilde{v}_F} \right)^2 \quad (\text{C9})$$

$$- \frac{1}{4} \left(\frac{\delta\omega_R}{\tilde{v}_F' k} \right)^2 - \frac{\pi^2}{24} \left(\frac{T}{\tilde{\epsilon}_F} \right)^2 + O(T^3).$$

This is the expansion we need.

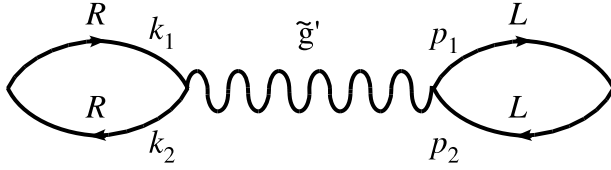


FIG. 1: Feynman diagram corresponding to the lowest order interaction correction to the density-density propagator.

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